Supplementary Materials for "Bayesian Low-Tubal-Rank Robust Tensor Factorization with Multi-Rank Determination"

1 Supplementary Experiments on Synthetic Data

In this section, we conduct supplementary experiments on synthetic tensors of size $I_1 \times I_2 \times I_3$ with $I_1 \neq I_2 \neq I_3$ to further validate the effectiveness of the proposed BTRTF method.

The synthetic data are generated as follows: Two factor tensors $\mathcal{U} \in \mathbb{R}^{I_1 \times R \times I_3}$ and $\mathcal{V} \in \mathbb{R}^{I_2 \times R \times I_3}$ are randomly generated with their elements independently drawn from the standard Gaussian distribution $\mathcal{N}(0,1)$. Then, the low-rank component is constructed by $\mathcal{X}_{gt} \in \mathbb{R}^{I_1 \times I_2 \times I_3} = \mathcal{U} * \mathcal{V}^{\dagger}$, and is further truncated by t-SVD to have $Rank_{\mathrm{m}}(\mathcal{X}_{gt}) = (R_{gt}^{(1)}, \ldots, R_{gt}^{(I_3)})$. The sparse component $\mathcal{S}_{gt} \in \mathbb{R}^{I_1 \times I_2 \times I_3}$ is generated by randomly selecting $\rho\%$ of the I^3 elements to be nonzero, whose values are uniformly drawn from $[-10, 10]$. The noise term $\mathcal{E} \in \mathbb{R}^{I_1 \times I_2 \times I_3}$ is generated by independently sampling its elements from $\mathcal{N}(0, \sigma^2)$. Finally, the observed tensor is constructed by $\mathcal{Y} = \mathcal{X}_{gt} + \mathcal{S}_{gt} + \mathcal{E}$.

Table 1 shows the recovery results of BTRTF on the synthetic data, where the rank error is defined as $Rank_{err} = \sum_{k=1}^{I_3} \frac{|\hat{R}^{(k)} - R_{gt}^{(k)}|}{I_3}$ and $\hat{R}^{(k)}$ is the estimated rank of the kth frontal slide. Table 2 shows the rank determination results of TCTF and BTRTF on the synthetic datasets with $\rho = 0\%$. As can be seen, BTRTF consistently performs well in both tensor recovery and multi-rank determination. In contrast, TCTF fails to determine the correct multi-rank and leads to large reconstruction error.

2 Supplementary Experiments on Image Denoising

We provide the normalized mean square error (NMSE) results for the image denoising experiments. The definition of NMSE is given by NMSE $= \frac{||\hat{\mathcal{X}} - \mathcal{X}_{gt}||_F^2}{||\mathcal{X}_{gt}||_F^2}$, where $\hat{\mathcal{X}}$ is the recovered tensor and $\mathcal{X}_{gt}$ is the ground truth. Table 3 shows the NMSE values on the BSD500 dataset. As can be seen, BTRTF obtains the smallest average NMSE, and outperforms the completing methods for 402 out of 500 images, which is consistent with the PSNR results shown in Fig. 2 (j) of our paper.

TABLE 1
Recovery results of BTRTF on the synthetic datasets.

$Rank_{\mathrm{m}}(\mathcal{X}_{gt}) = \{R, \overbrace{0.5R, \ldots, 0.5R}^{40}, \overbrace{R, \ldots, R}^{I_3-81}, \overbrace{0.5R, \ldots, 0.5R}^{40}\}$

$I_1 \times I_2 \times I_3$	R	ρ	σ^2	$Rank_{err}$	$\frac{\|\|\hat{\mathcal{X}}-\mathcal{X}_{gt}\|\|_F}{\|\|\mathcal{X}_{gt}\|\|_F}$	$\frac{\|\|\hat{\mathcal{S}}-\mathcal{S}_{gt}\|\|_F}{\|\|\mathcal{S}_{gt}\|\|_F}$
$100 \times 150 \times 200$	16	5%	0	0	1.24e−7	4.90e−6
			10^{-3}	0	9.24e−6	5.22e−4
		10%	0	0	1.93e−7	5.32e−6
			10^{-3}	0	9.56e−6	3.77e−4
		20%	0	0	5.01e−7	9.36e−6
			10^{-3}	0	1.03e−5	2.84e−4
$200 \times 150 \times 100$	16	5%	0	0	9.52e−8	2.29e−6
			10^{-3}	0	1.11e−5	6.25e−4
		10%	0	0	1.69e−7	2.86e−6
			10^{-3}	0	1.15e−5	4.43e−4
		20%	0	0	3.20e−7	3.74e−6
			10^{-3}	0	1.23e−5	3.21e−4

$Rank_{\mathrm{m}}(\mathcal{X}_{gt}) = \{0.5R, \overbrace{R, \ldots, R}^{40}, \overbrace{0.5R, \ldots, 0.5R}^{I_3-81}, \overbrace{R, \ldots, R}^{40}\}$

I	R	ρ	σ^2	$Rank_{err}$	$\frac{\|\|\hat{\mathcal{X}}-\mathcal{X}_{gt}\|\|_F}{\|\|\mathcal{X}_{gt}\|\|_F}$	$\frac{\|\|\hat{\mathcal{S}}-\mathcal{S}_{gt}\|\|_F}{\|\|\mathcal{S}_{gt}\|\|_F}$
$100 \times 150 \times 200$	16	5%	0	0	1.13e−7	4.14e−6
			10^{-3}	0	9.31e−6	5.46e−4
		10%	0	0	1.97e−7	5.05e−6
			10^{-3}	0	9.58e−6	3.93e−4
		20%	0	0	4.45e−7	7.75e−6
			10^{-3}	0	1.03e−5	2.91e−4
$200 \times 150 \times 100$	16	5%	0	0	1.20e−7	3.55e−6
			10^{-3}	0	1.10e−5	5.68e−4
		10%	0	0	2.04e−7	4.19e−6
			10^{-3}	0	1.14e−5	4.08e−4
		20%	0	0	4.31e−7	6.08e−6
			10^{-3}	0	1.23e−5	3.01e−4

TABLE 2
Rank determination results on the synthetic datasets with $\rho = 0\%$.

$Rank_{\mathrm{m}}(\mathcal{X}_{gt}) = \{R, \overbrace{0.5R, \ldots, 0.5R}^{40}, \overbrace{R, \ldots, R}^{I_3-81}, \overbrace{0.5R, \ldots, 0.5R}^{40}\}$

Method			TCTF		BTRTF	
$I_1 \times I_2 \times I_3$	R	σ^2	$Rank_{err}$	$\frac{\|\|\hat{\mathcal{X}}-\mathcal{X}_{gt}\|\|_F}{\|\|\mathcal{X}_{gt}\|\|_F}$	$Rank_{err}$	$\frac{\|\|\hat{\mathcal{X}}-\mathcal{X}_{gt}\|\|_F}{\|\|\mathcal{X}_{gt}\|\|_F}$
$100 \times 150 \times 200$	16	0	1.66	0.7069	0	2.84e−10
		10^{-3}	1.66	0.7074	0	8.94e−6
$200 \times 150 \times 100$	16	0	0.96	0.7069	0	2.85e−10
		10^{-3}	0.95	0.7068	0	1.09e−5

$Rank_{\mathrm{m}}(\mathcal{X}_{gt}) = \{0.5R, \overbrace{R, \ldots, R}^{40}, \overbrace{0.5R, \ldots, 0.5R}^{I_3-81}, \overbrace{R, \ldots, R}^{40}\}$

Method			TCTF		BTRTF	
$I_1 \times I_2 \times I_3$	R	σ^2	$Rank_{err}$	$\frac{\|\|\hat{\mathcal{X}}-\mathcal{X}_{gt}\|\|_F}{\|\|\mathcal{X}_{gt}\|\|_F}$	$Rank_{err}$	$\frac{\|\|\hat{\mathcal{X}}-\mathcal{X}_{gt}\|\|_F}{\|\|\mathcal{X}_{gt}\|\|_F}$
$100 \times 150 \times 200$	16	0	1.39	0.7069	0	2.49e−10
		10^{-3}	1.38	0.7074	0	8.96e−6
$200 \times 150 \times 100$	16	0	1.63	0.7069	0	3.15e−10
		10^{-3}	1.61	0.7068	0	1.07e−5

TABLE 3
NMSE values ($\times 10^{-3}$) on the 8 sample images (**Best**; Second best).

Image	1	2	3	4	5	6	7	8	Avg. on 500 images	#Best
RPCA	6.02	13.64	7.80	2.19	10.58	9.72	3.51	3.02	15.84	0
VBRPCA	11.38	38.01	15.18	4.88	26.76	27.11	10.03	8.82	39.81	0
BRTF	7.64	21.98	7.59	3.61	14.68	4.17	7.45	1.34	24.84	0
SNN	3.79	7.32	5.72	1.31	5.48	6.34	1.92	2.00	10.08	0
KDRSDL	3.93	4.21	5.27	1.57	7.14	3.22	3.76	1.37	6.94	32
TNN	2.40	5.69	3.70	1.00	3.01	4.15	1.35	1.43	6.37	66
BTRTF	**0.97**	**3.73**	**1.31**	**0.58**	**1.85**	**1.08**	**0.85**	**0.31**	**4.44**	**402**